MATH 2200

Makeup Midterm Exam I

September 30, 2009

NAME (please print legibly): _____

Your University ID Number: _____

Please complete all questions in the space provided. You may use the backs of the pages for extra space, or ask me for more paper if needed. Work carefully, and try to complete the problems you find easier before going back to the harder ones. Good luck!

QUESTION	VALUE	SCORE
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
TOTAL	105	

1. (10 points) Use the definition of the derivative as a limit to compute the derivative of

$$f(x) = x + \frac{1}{x}.$$

Please show your work. You will receive no credit for computing the derivative of $f(x)$ without using the definition of the derivative as a limit, but it might be a good idea to do this anyway just to check your work. (In any case, you will need this derivative formula in Problems 2 and 3.)

ANSWER: _____

2. (15 points) The point $(0, 4)$ is not on the graph of $y = x + 1/x$, but it is contained in exactly one tangent line to the graph. Find the value of a for which the tangent line to the graph of $y = x + 1/x$ through the point $(a, a + 1/a)$ contains the point $(0, 4)$.

ANSWER: _____

3. (10 points) Using your result from Problem 2, find the equation of the tangent of the tangent line to $y = x + 1/x$ which includes $(0, 4)$.

ANSWER: _____

3

4. (10 points) Please state the chain rule, then use the chain rule to find the derivative of

$$h(x) = (x^2 + 5x + 3)^{17}.$$

Please do **not** simplify your answer. Note: you will receive no credit for multiplying out $(x^2 + 5x + 3)^{17}$ by hand.

ANSWER: _____

5. (10 points) Please state the **quotient** rule, then find the derivative

$$\frac{\partial}{\partial y} h(x, y) = \frac{x^2 y + y^3}{y^2 + 2x + 3}.$$

using the quotient rule.

ANSWER: _____

6. (10 points) Recall that given a position function $s(t)$ for a particle on a straight line, the velocity $v(t)$ of the particle is given by $s'(t)$ and the acceleration of the particle is given by $a(t) = v'(t)$.

A car in a crash test is accelerated along a linear track at $2m/s^2$ for 5 seconds and then decelerated at $-4m/s^2$ until it strikes a barrier. The position function of the car is

$$s(t) = \begin{cases} t^2 & 0 \leq t \leq 5 \\ -2t^2 + At + B & t > 5 \end{cases}$$

Find the velocity $v(t)$ of the car. Note that your answer will have two cases, depending on whether $t > 5$ or $t \leq 5$.

ANSWER: _____

7. (10 points) There is only one set of values for A and B which make both the position function $s(t)$ and the velocity function $v(t)$ of Problem 6 continuous at time $t = 5$. Find these values.

ANSWER: _____

5

8. (10 points) The barrier in the crash-test of Problems 6 and 7 is located 33 meters from the start of the track at position $0 = s(0)$. Please find the time t at which the car impacts the barrier and the velocity $v(t)$ of the car at that point.

ANSWER: _____

9. (10 points) Find the derivative of $h(x) = \sqrt{1 + x^{1234}}$.

ANSWER: _____

10. (10 points) Find the limit
$$\lim_{\theta \to 0} \frac{\sin 6\theta}{5\theta}.$$

ANSWER: _____