# MATH 2260 

Midterm Exam I

February 18, 2015

NAME (please print legibly): $\qquad$
Your University ID Number: $\qquad$
Please complete all questions in the space provided. Draw a box around your final answer. You may use the backs of the pages for extra space, or ask me for more paper if needed. Work carefully, and neatly ( 2 points on every problem are given for clear presentation of your work or deducted for unclear, messy, or hard-to-understand work).

Try to complete the problems you find easier before going back to the harder ones. Good luck!

| QUESTION | VALUE | SCORE |
| ---: | ---: | ---: |
| 1 | 10 |  |
| 2 | 10 |  |
| 3 | 10 |  |
| 4 | 10 |  |
| 5 | 10 |  |
| 6 | 10 |  |
| 7 | 10 |  |
| 8 | 10 |  |
| 9 | 10 |  |
| TOTAL | 90 |  |

1. (10 points) Find the volume of the solid created by rotating the area (shown) between the curve $y=\sin x$, the $x$-axis, and the line $x=\pi / 2$ around the $y$-axis.

2. (10 points) Find the length of the portion of the graph of the function

$$
\begin{equation*}
f(x)=\frac{x^{2}}{2}-\frac{\ln x}{4} \tag{1}
\end{equation*}
$$

for $x$ from 1 to 3. (Set up the integral correctly: 6 points. Do the integral: 4 points).
3. (10 points) A particle at position $x$ experiences a force $F(x)=k / x^{2}$. Set up and evaluate the integral for the work done by $F(x)$ as the particle moves from $a$ to $b$. (Your answer will be in terms of $k, a$, and $b$.)
4. ( $\mathbf{1 0}$ points) Find the solution of the differential equation

$$
\frac{d y}{d x}=x^{2} \sqrt{y}
$$

where $y(1)=0$.
5. (10 points) Integrate

$$
\int \sin ^{5} x d x
$$

6. (10 points) Integrate

$$
\int \frac{\sqrt{9-x^{2}}}{x^{2}} d x
$$

7. ( 10 points) The table below gives some values for the function

$$
f(t)=\text { "air temperature at the Athens, GA weather station at time } t ",
$$

where temperature is given in degrees Farenheit and $t$ is measured in hours starting at 6 am on January 1, 2015 (time 0).

Use this data to estimate the average air temperature at the Athens weather station between 6 am and 6pm on January 1, 2015 using Simpson's rule. (Bonus +3 pts: Check your answer using the Trapezoid rule.)

The table has additional (blank) columns for your use.

| t | $\mathrm{f}(\mathrm{t})$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 39.92 |  |  |  |
| 3 | 41 |  |  |  |
| 6 | 41.36 |  |  |  |
| 9 | 39.92 |  |  |  |
| 12 | 37.22 |  |  |  |

8. (10 points) Evaluate the improper integral

$$
\int_{0}^{1} \frac{1}{x^{1 / 3}} d x
$$

9. (10 points) Bonus Credit (hard):


Prove that for any values of $p>0$ and $a>0$ the plate between $y=\frac{x^{2}}{4 p}$ and $y=a$ of uniform density has center of mass $\left(0, \frac{3}{5} a\right)$.

