Math 3500/H. Multivariable math.

Introduction.

Difficulty of course.

Gradescope.

Encouragement to work together.

Classroom culture.

Definition. $\mathbb{R}^n$ is the set of vectors

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}.$$

We can think of $\mathbb{R}^n$ as an n-dimensional space. We will learn to be comfortable in this setting, but don't need geometric intuition.

Definition. We let $\overrightarrow{AB}$ denote the vector from A (tail) to B (head).

and say

$$\overrightarrow{AB} = \begin{bmatrix} b_1 - a_1 \\ \\ b_n - a_n \end{bmatrix}$$

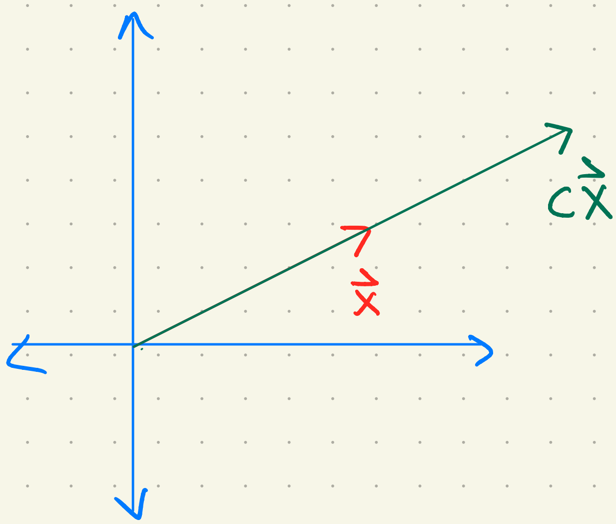
<u>Definition</u>. The length (or norm) of $\vec{x}$ is

$$\|\vec{x}\| = \sqrt{x_1^2 + \cdots + x_n^2}.$$

Note. There are other useful ways to measure length. This is the one we're familiar with from geometry class - it immediately implies Pythagorean thm.

<u>Definition</u>. A real number c is called a <u>scalar</u> and we define <u>scalar multiplication</u> by

$$c\vec{x} = \begin{bmatrix} cx_1 \\ \vdots \\ \vdots \\ cx_n \end{bmatrix}$$



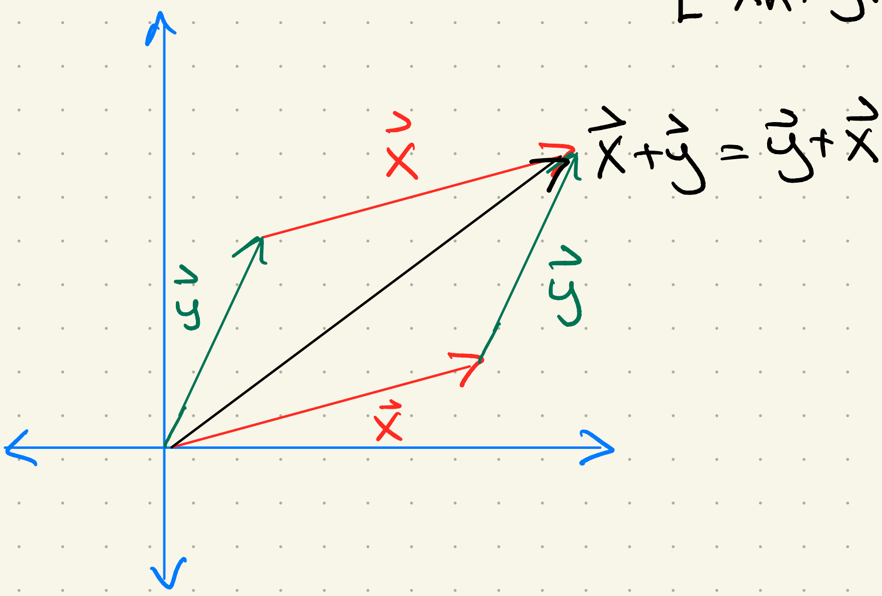
Exercise:

$$\|c\vec{x}\| = ?$$

<u>Definition</u>. We say $\vec{x}$ and $\vec{y}$ are <u>parallel</u> if there is some c so that $\vec{x} = c\vec{y}$ or $\vec{y} = c\vec{x}$.

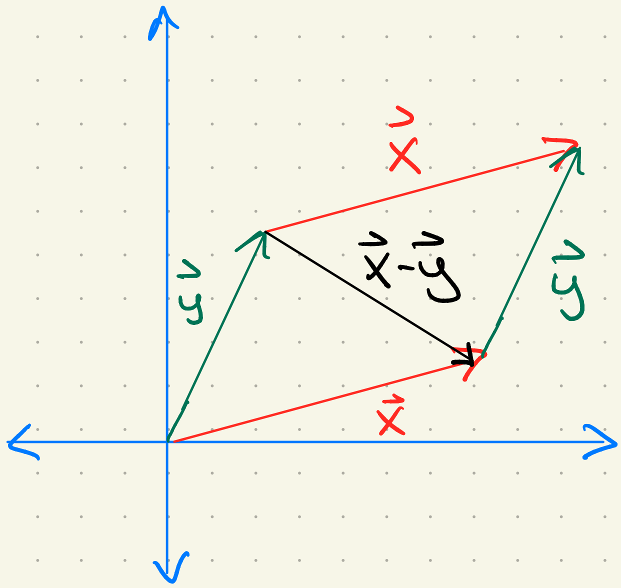
<u>Definition</u>. We define <u>vector addition</u> by

$$\vec{x} + \vec{y} = \begin{bmatrix} x_1 + y_1 \\ \vdots \\ x_n + y_n \end{bmatrix}.$$



We notice that

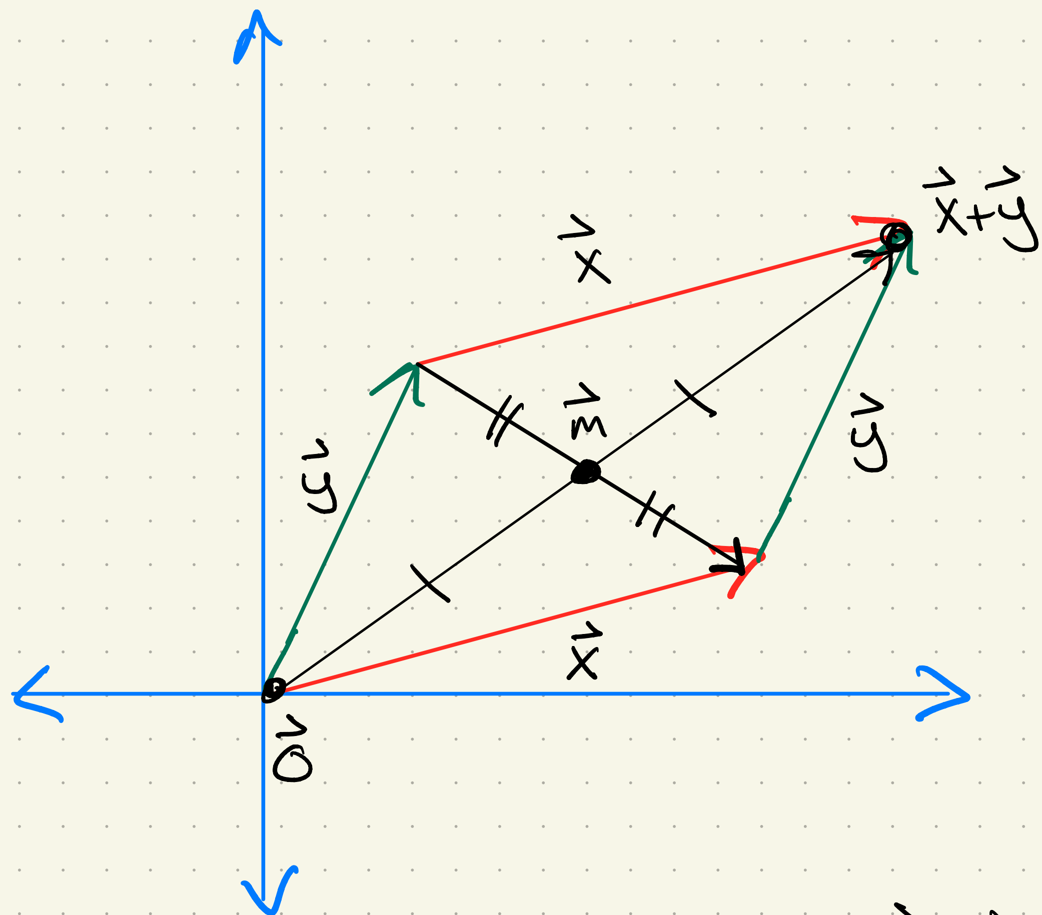
$$\vec{x} - \vec{y} = \vec{x} + (-1)\vec{y} = \begin{bmatrix} x_1 - y_1 \\ \\ x_n - y_n \end{bmatrix} = \overrightarrow{yx}$$



Vector algebra makes a <u>lot</u> of geometry simpler and clearer.

Fact. The midpoint of $\overrightarrow{AB}$ is $\frac{1}{2}\vec{A} + \frac{1}{2}\vec{B}$.

Proposition. The diagonals of a parallelogram bisect each other.



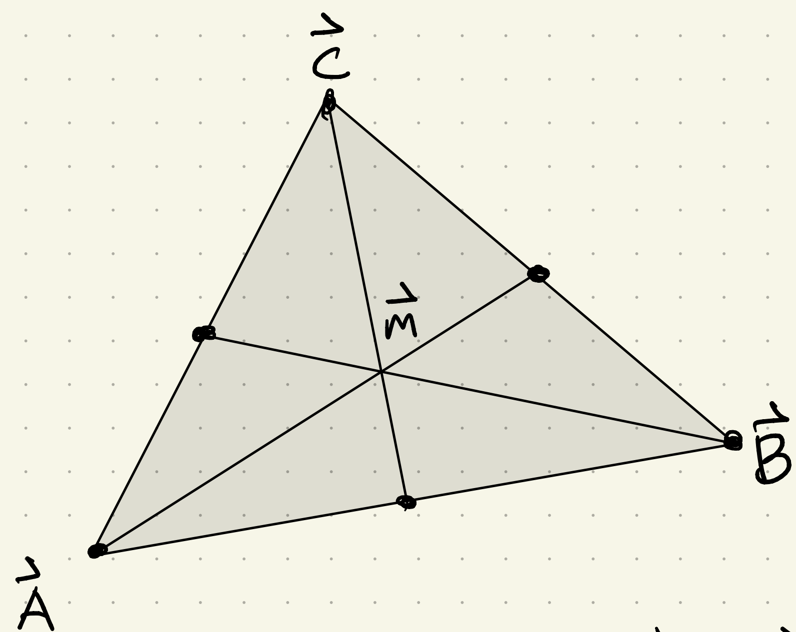
Proof. We will show that $\vec{m}$ is the midpoint of <u>both</u> $\overline{O(X+Y)}$ and $\overline{YX}$.

$$m = \underset{\vec{0}}{\cancel{\frac{1}{2}\vec{0}}} + \frac{1}{2}(\vec{x}+\vec{y}) = \frac{1}{2}\vec{x} + \frac{1}{2}\vec{y}.$$

$$m = \frac{1}{2}\vec{y} + \frac{1}{2}\vec{x} = \frac{1}{2}\vec{x} + \frac{1}{2}\vec{y}. \qquad \blacksquare$$

Definition. The line segment joining a vertex of a triangle to the midpoint of the opposite side is called a median.

Proposition. The medians of a triangle all intersect at a single point.



Proof. We will show that $\vec{m}$ is $2/3$ of the way from each vertex to the midpoint of the opposite side.

$$\vec{A} + \frac{2}{3}\left(\frac{1}{2}(\vec{B}+\vec{C}) - \vec{A}\right) = \vec{A} + \frac{1}{3}\vec{B} + \frac{1}{3}\vec{C} - \frac{2}{3}\vec{A}$$
$$= \frac{1}{3}\vec{A} + \frac{1}{3}\vec{B} + \frac{1}{3}\vec{C}$$

$$\vec{B} + \frac{2}{3}\left(\frac{1}{2}(\vec{A}+\vec{C}) - \vec{B}\right) = \vec{B} + \frac{1}{3}\vec{A} + \frac{1}{3}C - \frac{2}{3}\vec{B}$$
$$= \frac{1}{3}\vec{A} + \frac{1}{3}\vec{B} + \frac{1}{3}\vec{C}.$$

$$\vec{C} + \frac{2}{3}\left(\frac{1}{2}(\vec{A}+\vec{B}) - \vec{C}\right) = \vec{C} + \frac{1}{3}\vec{A} + \frac{1}{3}B - \frac{2}{3}\vec{C}$$
$$= \frac{1}{3}\vec{A} + \frac{1}{3}\vec{B} + \frac{1}{3}\vec{C}.$$

$\boxtimes$

<u>Porism</u>. The point of intersection of the medians of a triangle is the vector average of the vertices.