MATH 2200

Final Exam

December 15, 2004

NAME (please print legibly): ____________________

Your University ID Number: ____________________

Please complete all 20 questions in the space provided. You may use the backs of the pages for extra space, or ask me for more paper if needed. Work carefully, and try to complete the problems you find easier before going back to the harder ones. Good luck!

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	15	
5	10	
6	10	
7	15	
8	10	
9	10	
10	10	
11	10	
12	10	
13	20	
14	10	
15	10	
16	10	
TOTAL	180	

1. (10 points) This question has three parts:

1. State the quotient law **for limits**.
   (Remember that it is different from the quotient rule for derivatives.)

2. State the product law **for limits**.
   (Again, remember that it is different from the product rule for derivatives.)

3. If it exists, evaluate the limit
   $$\lim_{x \to 4} \frac{x^2 - 16}{2 - \sqrt{x}}.$$
   You may use any method you like (including L'Hopital's rule).

ANSWER: ____________________

2. (10 points) If it exists, find the one-sided limit

$$\lim_{x \to 3^+} \frac{\sqrt{x^2 - 6x + 9}}{3 - x}.$$

ANSWER: ____________________

3. (10 points) This question has two parts:

1. Define **continuity at a point** for a function $f(x)$ at a point a.

2. State the **intermediate value theorem** for continuous functions on a closed interval.

4. (15 points) This question has three parts:

1. Write down the **definition of the derivative as a limit** for a function $f(x)$.

2. Use that definition (and the limit laws, but *not* L'Hopital's rule) to find the derivative of

$$f(x) = \frac{x+1}{x-1}.$$

ANSWER: ____________________________

3. Compute $f'(x)$ using the quotient rule for derivatives (and check the result against your answer in part 2).

ANSWER: ____________________________

5. (10 points) Compute the derivative of

$$f(t) = (t^2 + 1)(t^3 + t^2 + 1) \tag{1}$$

using the product rule. Do **not** simplify your answer.

ANSWER: ______________________________

6. (10 points) This question has two parts:

1. State **the chain rule** for the derivative of the composition $f(g(x))$ of functions $f(x)$ and $g(x)$.

2. Let $h(x) = \sin(x^3)\cos(x^3)$. Find functions $f(x)$ and $g(x)$ so that $h(x) = f(g(x))$. Then use the chain rule to compute $h'(x)$.

ANSWER: ______________________________

7. (15 points) Find the **maximum** and **minimum** values of the function

$$f(x) = x + \frac{4}{x}$$

on the closed interval $[1, 4]$. Note: we are asking for the *values* (or "y values") of the max and min instead of their *locations* (or "x values").

ANSWER: ______________________

8. (10 points) Using implicit differentiation, find the equation of the tangent line to the curve

$$xy = 6e^{2x-3y}$$

at the point $(3, 2)$. Please express your answer in point-slope form.

ANSWER: ____________________________

9. (10 points) This problem has two parts:

1. Fill in the blank to write down the **linear approximation** to a function $f(x)$ at a point a.

   $f(x) \approx$

2. Use linear approximation to estimate the numerical value of $e^{1/10}$.

ANSWER: ____________________________

10. (10 points) The width of a certain rectangle is always half its length. At what rate is the **area** of the rectangle increasing if the width is 10 cm and the width is increasing at 0.5 cm/sec?

ANSWER: ____________________

11. (10 points) Find the intervals on which the function

$$f(x) = \frac{x}{x+1}$$

is **increasing** and **decreasing**.

ANSWER: ____________________________

12. (10 points) Find all the critical points of

$$f(x) = xe^{-2x}$$

and classify each as a **local maximum**, **local minimum**, or **saddle point**.

ANSWER: ____________________________

13. (20 points) Sketch the best graph you can of

$$f(x) = \frac{1}{e^x + e^{-x}}.$$

Be sure to indicate any

1. Horizontal or vertical asymptotes.
2. Critical points and increasing/decreasing intervals.
3. Intervals on which the graph is concave up/concave down.

ANSWER: ______________________

14. (10 points) This problem has two parts:

1. State **L'Hopital's rule**.

2. Use L'Hopital's rule to compute

$$\lim_{x\to 0} \frac{\ln(1+x)}{x}.$$

ANSWER: ____________________

3. (5 bonus points) Compute

$$\lim_{x\to\infty} \frac{2^x}{3^x}.$$

ANSWER: ____________________

15. (10 points) This problem has two parts:

1. Compute
$$\int (5\cos(10x) - 10\sin(5x))\,\mathrm{d}x.$$

ANSWER: ________________________

2. Differentiate your answer above. Do you get the expected answer?

ANSWER: ________________________

16. (10 points) Solve the **initial value problem**

$$\frac{dy}{dx} = \frac{x}{y}, \qquad y(3) = 5.$$

for the function $y(x)$.

ANSWER: ____________________________