

MATH 2250

Midterm Exam II

October 26, 2015

NAME (please print legibly): _____

Your University ID Number: _____

Please complete all questions in the space provided. You may use the backs of the pages for extra space, or ask me for more paper if needed. This exam will be graded on:

- Correctness of computations.
- Clarity of explanation of procedure.
- Correctness of procedure.

A correct answer obtained using an incorrect or poorly explained procedure will not be graded for full credit. Please feel free to write as much as you like. Work carefully, and try to complete the problems you find easier before going back to the harder ones. Good luck!

Remember that you are **required** to have a non-graphing calculator to complete the exam. Remember also that smartphone (or computer, or other device) use is **prohibited** on this exam, regardless of what you use it for.

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
TOTAL	80	

1. (10 points) Find the limit

$$\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$$

using L'Hospital's rule.

ANSWER: _____

2. (10 points) It is an odd fact about prices that they generally go up. And when they go up, they all go up together. Indeed, the price of apples (A) and the price of grapefruit (G) are related to the price of carrots (C) by the simple equation:

$$\sqrt{AG} = 2C.$$

Suppose that the price of apples is \$3 and is rising at \$0.25/yr, while the price of carrots is \$1.5 and is rising at \$0.15/yr. What is the price of grapefruit? And at what rate is it rising (or falling)?

ANSWER: _____

3. (10 points) A certain rocket engine produces $y = x^2e^x$ Newtons of thrust with a fuel input of x liters/sec of hydrazine.

- A fuel pump provides 2 ± 0.05 liters/sec of hydrazine. Find the *nominal thrust* $y(2)$ and use linear approximation to estimate the variation in thrust Δy caused by the variation Δx in fuel flow.

ANSWER: _____

- Is the thrust function concave up or concave down on the interval $[1, 5]$?

ANSWER: _____

- (Harder) Sketch a function with the concavity (up or down) that you found above, and draw a tangent line to the function. Conclude from the drawing whether the tangent line approximation to the function is generally an overestimate of the function (too high) or an underestimate of the function (too low).

ANSWER: _____

4. (10 points) Some webpage on the Internet claims that the longest home run ever recorded in Major League Baseball was hit by Babe Ruth in 1919 and followed the trajectory

$$y = x - 0.0017x^2$$

where x represents the distance (in feet) from home plate, and y represents the height (in feet) of the ball above the ground.

Find **the greatest height reached by the ball** and **how far from home plate the ball landed**.

ANSWER: _____

5. (10 points) Find the shortest path from the point $(-4, 2)$ to the point $(6, 3)$ which touches the x -axis at some point $(x, 0)$. (Hint: You can assume that the path is a straight line from $(-4, 2)$ to $(x, 0)$, followed by a straight line from $(x, 0)$ to $(6, 3)$.)

ANSWER: _____

6. (10 points) Presidential candidate Senator Sanders was polling at 5% on March 4, 2015 and on April 4, 2015 his support had risen to 6.6%: an average rate of change of 1.6%/month. Use linear approximation to estimate the Senator's support on June 4, 2015.

ANSWER: _____

Better polling data shows that 1.1% of the rise in support for Senator Sanders occurred in the last two weeks of March. That is, the rate of change of the rate of change of the Senator's support can be estimated as $1\%/month^2$. Use that data to update your estimate for the Senator's support on June 4, 2015 using quadratic approximation.

ANSWER: _____

According to HuffPost, Senator S had 12.5% support in the polls on June 4. Which estimate was better?

ANSWER: _____

7. (10 points) (The Laffer Curve, Theory, Harder). Consider the total revenue $M(t)$ raised by taxing stock market earnings (capital gains) at a rate of t (percent). Suppose that $M(t)$ is a continuous and differentiable function on the range $[0, 100]$. If the tax rate is 0 percent, the revenue raised is 0 because no taxes are collected. If the tax rate is 100 percent, the revenue raised is also 0 because nobody will bother to invest if they know government will collect any money they make.

Explain why calculus tells us that there is some tax rate t_0 which raises the most revenue. What is $M'(t)$ at this point?

ANSWER: _____

Explain why calculus tells us that there is some tax rate t_l where raising the tax rate slightly will raise more revenue, and some tax rate t_h where lowering the tax rate slightly will raise more revenue.

ANSWER: _____

Does this analysis tell you whether raising the current capital gains tax rate from 28% to 29% will produce more or less tax revenue? Why or why not?

ANSWER: _____

8. (10 points) (Extra credit). The area A of a triangle may be written in terms of the lengths of the three sides of the triangle a , b , and c by the formula

$$16A^2 = 4a^2b^2 - (a^2 + b^2 - c^2)^2.$$

where c is between $a - b$ and $b - a$. Suppose that sidelengths a and b are constant. Find the value of c so that $dA/dc = 0$. Use the first or second derivative test to decide whether this value is a maximum or a minimum.

ANSWER: _____