Problem Session 1.

Use the arithmetic-geometric mean inequality to show that among all triangles of perimeter p, the equilateral one has the most area.

Show that for positive x, y, α, β

$$x^{\alpha} y^{\beta} \leq \frac{\alpha}{\alpha+\beta} x^{\alpha+\beta} + \frac{\beta}{\alpha+\beta} y^{\alpha+\beta}$$

and hence $x^{2004} y + x y^{2004} \leq x^{2005} + y^{2005}$.

(Harder) Prove $a+b+c \leq \frac{a^3}{bc} + \frac{b^3}{ac} + \frac{c^3}{ab}$.