

Elementary operations. Interchange, multiply, linear combine.

Prop. Elementary operations don't change solutions.

Augmented matrix.

Example.  $3x_1 - 2x_2 + 9x_4 = 4$   
 $2x_1 + 2x_2 - 4x_4 = 6$

Definitions. Leading entry, echelon form, reduced echelon form

pivot - no leading entry in same column

free - not pivot.

Pivot variables are determined by free variables.

Ex.  $x_1 + x_2 + 3x_3 - x_4 = 0$   
 $-x_1 + x_2 + x_3 + x_4 + 2x_5 = -4$   
 $x_2 + 2x_3 + 2x_4 - x_5 = 0$   
 $2x_1 - x_2 + x_4 - 6x_5 = 9.$

Gaussian elimination.

Swap rows to put nz in top left

Kill all entries in 1st column.

Continue.

Definition. Reduced row echelon

Theorem. The reduced row echelon form of  $M$  is unique.

Ex. Write

$$b = \begin{bmatrix} 4 \\ 3 \\ 1 \\ 2 \end{bmatrix}$$

as a linear combination

of  $v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$

$$v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$