Math 4500/6500 Homework #9

This homework assignment covers our notes on Simpson's Rule and one-variable minimization.

1. If $f(x)$ is unimodal and continuous on $[a, b]$, how many local maxima may $f(x)$ have on $[a, b]$? Prove it.

2. Carry out four steps of the Fibonacci search algorithm using $\epsilon = \frac{1}{4}$ to find:

   (1) the minimum of $F(x) = 2x^3 - 9x^2 + 12x + 2$ on $[0, 3]$
   (2) the minimum of $F(x) = 2x^3 - 9x^2 + 12x$ on $[0, 2]$

3. Now use Brent's method for four steps on the same functions

   (1) the minimum of $F(x) = 2x^3 - 9x^2 + 12x + 2$ on $[0, 3]$
   (2) the minimum of $F(x) = 2x^3 - 9x^2 + 12x$ on $[0, 2]$

   and compare the results.

4. We mentioned in class that finding the minimum of a function by trying to solve for points where $f'(0)$ with a rootfinding code is likely to be a bad idea. Actually try this method on the function $f(x) = \cos^4 x$ (and compare to Brent's method) on the interval $[1, 3]$.