# Math 4510/6510 Homework #2

1. Find coefficients $a$, $b$, $c$, $d$, and $e$ so that
$$x(t+h) = a\,x(t) + b\,x(t-h) + h\left[c\,x'(t+h) + d\,x''(t) + e\,x'''(t-h)\right]$$
holds for polynomials $x(t)$ of as high a degree as possible.

2. Why do the coefficients in the Adams-Bashforth formulas add up to 1?

3. Determine the numerical value of
$$2\pi \int_4^5 \frac{e^x}{x}\,dx$$
in three ways: numerical integration, solving the ODE numerically, and integrating symbolically and evaluating the exact formula. Is it better to numerically integrate? Or numerically solve the ODE?

4. The second-order Adams-Bashforth-Moulton method is given by
$$\tilde{x}(t+h) = x(t) + \frac{h}{2}\left[3f(t,x(t)) - f(t-h,x(t-h))\right]$$
$$x(t+h) = x(t) + \frac{h}{2}\left[f(t+h,\tilde{x}(t+h)) + f(t,x(t))\right]$$
The approximate single-step error is
$$\epsilon \simeq \frac{1}{6}\left|x(t+h) - \tilde{x}(t+h)\right|$$
Using *Mathematica*, write and test an adaptive procedure for solving an ODE (you can pick the ODE) which uses this method and uses $\epsilon$ to monitor convergence and adjust step size accordingly.

5. Explain how to solve the system
$$x_1'(t) = x_1(t)e^t + \sin t - t^2$$
$$x_2'(t) = [x_2(t)]^2 - e^t + x_2(t)$$
$$x_1(1) = 2, \quad x_2(1) = 4$$
using only a solver for equations in the form $x'(t) = f(t,x(t))$ for a single function $x(t)$.

6. Convert the differential equation
$$x'''(t) = t + x + 2x' + 3x''$$
$$x(1) = 3$$
$$x'(1) = -7$$
$$x''(1) = 4$$
into a first-order system of ODE.

7. Solve Airy's equation:
$$x'' = t\,x, \quad x(0) = 0.355028053887817, \quad x'(0) = -0.258819403792807$$
using your own code in *Mathematica*. You can check your code with $x(4.5) = 0.0003302503$, which is correct.