Combinations

Definition. $\binom{n}{j}$ is the number of distinct subsets of a set of n elements with j members.

Example. $\binom{3}{0}, \binom{3}{1}, \binom{3}{2}, \binom{3}{3}$, sum is 2^n.

Theorem. If $n, j \in \mathbb{Z}$, $0 < j < n$, $\binom{n}{j} = \binom{n-1}{j} + \binom{n-1}{j-1}$.

Proof. ~~Se~~ Fix u. $\binom{n-1}{j}$ = # j elt sets w/o u, $\binom{n-1}{j-1}$ = # j elt sets w/u

Pascal's triangle.

Theorem. $\binom{n}{j} = \frac{(n)_j}{j!} = \frac{n!}{j!(n-j)!}$

Poker hands. There are $\binom{52}{5} = 2{,}598{,}960$ hands.

Find prob of Royal flush, ~~fl~~ straight flush, four of kind, full house, flush, straight, three of kind, two pairs, pair, high card ace.

Example. 4 of a kind. 13 cards to have 4 of $\times$ 48 cards left

Full house. 13 choices for the triple card $\times \binom{4}{3}$ ways to pick them from 4 suits $\times$ 12 choices for double card $\times \binom{4}{2}$ ways to pick them from 4 suits.

Definition. Bernoulli trials are a sequence of n chance experiments, each with two outcomes called success and failure. The probability of success is p and failure is $q = 1-p$.

Definition. $b(n,p,j)$ = probability of exactly j successes in n trials w/success prob p.

Theorem. $b(n,p,j) = \binom{n}{j} p^j q^{n-j}$

Demonstration. DiscretePlot[PDF[BinomialDistribution]]

Backsolving for p from data. Suppose we conduct 1000 trials and get 600 successes. We know that these were bernoulli trials with different p.

$$P(b(n,p,j) = 600) = \frac{600 \cdot \binom{n}{j} p^j (1-p)^j}{n+1}$$

Demonstration. Show continuous pdf, integrate over regions, mention Bayesian p.o.v.

Inclusion exclusion.

$$0 = (1-1)^K = \sum_{j=0}^{K} \binom{K}{j} (-1)^j = \overbrace{\binom{K}{0}}^{1} - \overbrace{\sum_{j=1}^{K} \binom{K}{j} (-1)^j}^{1}$$