

# MATH 2250

## Midterm Exam I

June 13, 2014

**NAME (please print legibly):** \_\_\_\_\_

**Your University ID Number:** \_\_\_\_\_

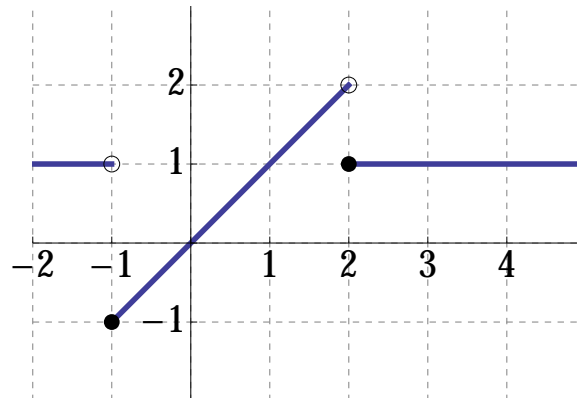
Please complete all questions in the space provided. You may use the backs of the pages for extra space, or ask me for more paper if needed. This exam will be graded on:

- Correctness of computations.
- Clarity of explanation of procedure.
- Correctness of procedure.

A correct answer obtained using an incorrect or poorly explained procedure will not be graded for full credit. Please feel free to write as much as you like. Work carefully, and try to complete the problems you find easier before going back to the harder ones. Good luck!

QUESTION	VALUE	SCORE
1	15	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
<b>TOTAL</b>	<b>75</b>	

1. (15 points) Consider the function  $f(x)$  whose graph is pictured below:



Find the following limits, or write DNE (and a sentence explaining why) if they do not exist. **Read carefully: some of the limits below are one-sided.**

$$\lim_{x \rightarrow 0} f(x) =$$

ANSWER: \_\_\_\_\_

$$\lim_{x \rightarrow 2} f(x) =$$

ANSWER: \_\_\_\_\_

$$\lim_{x \rightarrow (-1)^+} f(x) =$$

ANSWER: \_\_\_\_\_

**2. (10 points)** State the definition of the derivative as a limit, and use it to compute the derivative of the function  $f(x) = 2x + 3$  (by doing the limit). **This problem is graded on your work (the answer is easy to get using the differentiation rules) so write out the limit problem carefully.**

ANSWER: \_\_\_\_\_

Now compute the derivative of  $f(x) = 2x + 3$  the easy way and verify that it matches the limit above.

ANSWER: \_\_\_\_\_

3. (10 points) Find the derivative

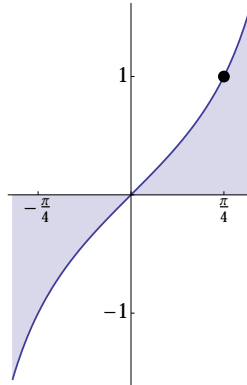
$$\frac{d}{dx} \left( \frac{\sin x}{x^2 + 3x} \right)^2.$$

For 5 bonus points, use **logarithmic differentiation** to simplify your work.

DO NOT SIMPLIFY ANYTHING.

ANSWER: \_\_\_\_\_

4. (10 points) Consider the function  $f(x) = \tan x$ , whose graph is shown below:



The point  $(\pi/4, 1)$  is marked on the graph.

1. Sketch the tangent line to this function at this point on the graph above.
2. Find the equation for this tangent line in point-slope form.

ANSWER: \_\_\_\_\_

**5. (10 points)** Differentiate both sides of the equation (known to you from Anki cards):

$$\sin 2x = 2 \sin x \cos x$$

to get a different trigonometric identity (also an Anki card). What is this second identity?

ANSWER: \_\_\_\_\_

**6. (10 points)** The volume of a spherical balloon (in cubic feet) is given as a function of the radius ( $r$ , in feet) of the balloon by

$$V(r) = \frac{4}{3}\pi r^3$$

Suppose that air is blown into the balloon and the (changing) radius is given as a function of time ( $t$ , in seconds) by the equation

$$r(t) = e^t.$$

At time  $t = 1$  sec, find the **rate of change** of the volume of the balloon, **including units**.

ANSWER: \_\_\_\_\_

**7. (10 points)** Use logarithmic differentiation to find the derivative of

$$f(x) = (\sin x)^{(\tan x)}$$

Again, **do not simplify**.

ANSWER: \_\_\_\_\_